

扫描有限元法求解二元喷管的 三维可压粘性流场

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【摘要】 发展了一种二元喷管流场计算的三维 N-S 方程扫描有限元方法,给出了 N-S 方程中二阶导数项新的离散方法,以及更完善、简洁的扫描有限元方程的表达式。开发了相应的计算程序,用所发展的方法对二维平面收扩喷管、两种大宽高比的二元喷管及其圆变方转接段的三维粘性流场进行了计算,计算结果表明本方法具有较宽广的适用范围,计算结果良好。

主题词: 有限元法 喷管 三维流动

分类号: O357 O242.21

1 前言

扫描有限元法在保持了传统有限元的优点以外,又不必构造适用于整体的泛函和集合单元方程,而是直接应用控制方程在每一单元上建立有限元方程,并逐块求解,从而避免了求解一个包括全部节点未知量的大型非线性方程组,故可以方便地求解 N-S 方程或欧拉方程^[1]。二元喷管使飞行器的机动性能得到改善^[2,3]。然而,它需要一圆转方过渡段与涡轮相衔接,过渡段的型面变化较剧烈,它内部的流动也较为复杂。扫描有限元法在求解这样的复杂可压粘性流场,显示出了它独到的优越性。

2 扫描有限元法

在有限元分析中,一个重要的问题是选择元素和在元素内定义合适的插值函数,本文选用 27 节点六面体单元,插值函数为:

$$Q(\alpha, \beta, \gamma) = (a_1\alpha^2 + a_2\alpha + a_3)(b_1\beta^2 + b_2\beta + b_3)(c_1\gamma^2 + c_2\gamma + c_3) \quad (1)$$

这样选取的插值函数具有对称性,在边界上 C^0 连续,并且每个方向上都有常数项、一次项、二次项来表征位移、速度、加速度或者粘性项的变化。根据插值函数在各节点的性质,可求得各节点的插值函数:

$$N_i = f(\alpha, \beta, \gamma, \alpha_i, \beta_i, \gamma_i) \quad (i = 1, \dots, 27) \quad (2)$$

进行等参变换:

$$x = \sum_{i=1}^{27} x_i N_i, \quad y = \sum_{i=1}^{27} y_i N_i, \quad z = \sum_{i=1}^{27} z_i N_i \quad (3)$$

这样物理域转换为标准计算域。此时给定 α, β, γ 值,即可求出:

$$\begin{aligned} \left[\frac{\partial N_i}{\partial \alpha} \quad \frac{\partial N_i}{\partial \beta} \quad \frac{\partial N_i}{\partial \gamma} \right]^T &= J^{-1} \left[\frac{\partial N_i}{\partial \alpha} \quad \frac{\partial N_i}{\partial \beta} \quad \frac{\partial N_i}{\partial \gamma} \right]^T \\ \left[\frac{\partial^2 N_i}{\partial \alpha^2} \quad \frac{\partial^2 N_i}{\partial \alpha \partial \beta} \quad \frac{\partial^2 N_i}{\partial \alpha \partial \gamma} \quad \frac{\partial^2 N_i}{\partial \beta^2} \quad \frac{\partial^2 N_i}{\partial \beta \partial \gamma} \quad \frac{\partial^2 N_i}{\partial \gamma^2} \right]^T &= J^{-1} \left[\frac{\partial^2 N_i}{\partial \alpha^2} \quad \frac{\partial^2 N_i}{\partial \alpha \partial \beta} \quad \frac{\partial^2 N_i}{\partial \alpha \partial \gamma} \quad \frac{\partial^2 N_i}{\partial \beta^2} \quad \frac{\partial^2 N_i}{\partial \beta \partial \gamma} \quad \frac{\partial^2 N_i}{\partial \gamma^2} \right]^T \end{aligned} \quad (4)$$

其中, $N_{ix_j} = \partial N_i / \partial x_j$, x_j 为 x, y, z , J 为雅可比矩阵。各流场变量 φ 及导数可利用下式求出:

$$\varphi = \sum_{i=1}^{27} \varphi_i N_i, \quad \frac{\partial \varphi}{\partial x_j} = \sum_{i=1}^{27} \varphi_i \frac{\partial N_i}{\partial x_j}, \quad \frac{\partial^2 \varphi}{\partial x_j \partial x_k} = \sum_{i=1}^{27} \varphi_i \frac{\partial^2 N_i}{\partial x_j \partial x_k} \quad (5)$$

扫描有限元法要求将方程写成积分形式, 时间相关三维直角坐标系强守恒形式的 N-S 方程可写为:

$$\iiint_V \frac{\partial U}{\partial t} dx dy dz = \iiint_V \left(- \frac{\partial F}{\partial x} - \frac{\partial G}{\partial y} - \frac{\partial E}{\partial z} + \frac{\partial H}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial S}{\partial z} \right) dx dy dz \quad (6)$$

其中, U, F, G, E, H, Q, S 为向量表达式^[4]。求得导数并进行等参变换, 用高斯积分将其转化为代数方程, 完成空间离散:

$$\Delta U = \frac{1}{V} \sum W_k W_j W_l [J(\alpha, \beta, \gamma_k) \left(- \sum_{i=1}^{27} F_i \frac{\partial N_i}{\partial x} - \sum_{i=1}^{27} G_i \frac{\partial N_i}{\partial y} - \sum_{i=1}^{27} E_i \frac{\partial N_i}{\partial z} F_{visc}^i \right)] \quad (7)$$

其中, V 是单元体体积, F_{visc}^i 是适合于扫描有限元离散的第 i 节点的粘性通量, 紊流粘性系数由 B-L 湍流模型求出。 W_l, W_j, W_k 为高斯积分权因子, α, β, γ_k 为积分基点的坐标, 3 个方向所取的点数不同, 则 i, j, k 前取不同的值, 本文所取高斯积分点为中心节点。采用 Jameson^[4] 多步龙格—库塔法进行时间推进, 完成对方程的时间离散, 并加入二阶及四阶人工粘性项 D :

$$U^{(m)} = U^{(0)} + \Delta t / (5 - m) (\Delta U + D)^{(m-1)} \quad (m = 1, \dots, 4) \quad (8)$$

涡轮出口的圆截面转变为喷管的矩形出口截面, 这一变化过程是由圆→超椭圆→矩形等面积变化而来。使用 O 型网格在矩形截面网格分布不合理, 而用一般的代数方法生成的 H 型网格在圆及椭圆截面壁面正交性又不好。本文用 TTM 网格生成法先生成正交性较好的粗网, 然后在壁面处再进行加密。

3 算 例

图 1 所示是一个二元收扩喷管的计算结果。计算了落压比为 2 和 4 两种工况(图中 Δ 落压比 2, $\circ$ 落压比 4), 前者喷管出口附近的壁面已产生分离。由壁面压力分布曲线可以看出计算结果与实验结果^[5] 相当吻合。图 2, 3 所示分别为宽高比 B 为 10 和 4 的二元喷管及其圆转方转接段三维流场的计算结果。在图 2 中, 由等值线可以看出在 z 方向的突扩处有较大的分离区, 这一分离区使流通面积急剧减小, 以至气流达到超音速, 并在出口处出现激波, 这与实验很吻合, 实验中喷管壁面被烧蚀。由横截面矢量图可以看出由粘性影响而产生的二次流及涡。在图 3 中, 分离区较小, 而且分离后再附。在出口壁面附近有一微小分离区。由横截面矢量图, 可以看出喷管内有二次流, 在出口处有涡。出口处的涡正是减少红外辐射所需要的, 它将周围的冷气迅速卷吸入射流, 使射流温度迅速降低, 从而降低红外辐射, 增强飞行器的隐身性。

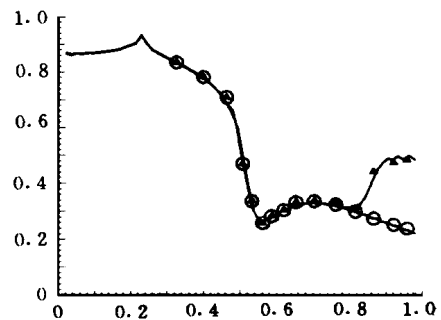


图 1 收扩喷管壁面压力分布曲线

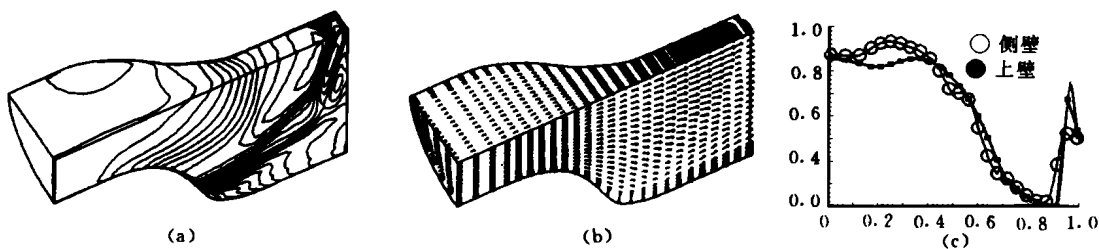


图 2 宽高比 $B=10$ 二元喷管计算结果

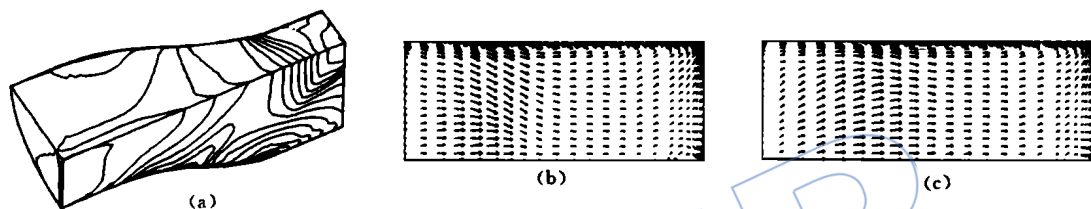


图 3 宽高比 $B=4$ 二元喷管计算结果

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straight blade cascades.

Key Words Compressors Cascade flow Solidity

THREE- DIMENSIONAL FLOW FIELD INSIDE A COMPRESSOR ROTOR TIP REGION

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ABSTRACT

An experimental study of the three- dimensional flow field inside the tip region of a low- speed, large- scale axial- flow compressor rotor has been completed. With the help of a rotating four- coordinate probe traverse mechanism, the measurements were conducted with a five- hole conical probe in both design condition and near stall condition. The results indicate that the tip leakage vortex plays a decisive role in the tip region flow in the design condition, leading to substantial flow loss and blockage in the passage. In near stall condition, flow separation occurs on the blade suction surface while radial migration of the boundary layer on the suction surface is strong, which makes more low- energy fluid accumulate at the corner between the casing annual wall and the suction surface. A corner vortex is induced by the tip leakage flow, and the low- energy fluid migrates to the pressure side due to the effect of the leakage vortex. Strong flow mixing among the corner vortex, the leakage vortex, the boundary layer on the casing wall and the main flow occurs at median part of the passage, which leads to substantial deficiencies of relative total pressure and relative velocity.

Key Words Compressors Rotors Three- dimensional boundary layer

SWEEP FINITE ELEMENT SOLVER FOR 3- D VISCOUS COMPRESSIBLE FLOW IN RECTANGULAR NOZZLE

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ABSTRACT

A numerical anlysis method of three- dimensional compressible viscous flow has been developed for a rectangular exhaust nozzle in which the shape of the circular- to- rectangular duct changes abruptly and its flowfield is very complex. Therefore, the sweep finite element method proposed by the second author in 1983 is employed since the governing equations are directly used to establish the element equation. In this paper, a hexahedral element of 27 nodes is adopted for spacial discretization, and applied to a C- D nozzle flowfield and several transition duct and nozzle flow fields. The results are in good agreement with the experimental data, and indicate the effectiveness and accuracy of the method.

Key Words Finite element methods Jet nozzles Three- dimensional flows

EXPERIMENTAL STUDY ON PIN FIN ARRAY FLOW FIELD

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ABSTRACT

An experimental study was performed to investigate the staggered pin fin array flow field in the specially constructed small wind tunnel. The experimental result gives the detailed distributions of the flow field parameters. The analysis reveals that the